

Massachusetts Institute of Technology

Department of Physics

Course: 8.701 – Introduction to Nuclear and Particle Physics
Term: Fall 2020
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Discussion Problems

from recitation on **September 3rd, 2020**

Problem 1: Gamma Factor for LEP Electron

At LEP @ CERN, electrons and positrons were accelerated to 100 GeV. How large was γ ?

- We know that:

$$E_e = \gamma m_e c^2 = 100 \text{ GeV},$$

and:

$$m_e c^2 \approx 0.5 \text{ MeV} = 5 \times 10^{-4} \text{ GeV}.$$

So,

$$\gamma = \frac{100 \text{ GeV}}{5 \times 10^{-4} \text{ GeV}} = \boxed{2 \times 10^5}.$$

Problem 2: Splitting the Deuteron

How much energy do we need to split a proton and neutron (deuteron)?

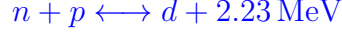
- In the atomic mass unit, $1 u = 1.66 \times 10^{-27} \text{ kg}$, the masses of the relevant particles are:

$$m_d = 2.01355 u,$$

$$m_p = 1.00728 u,$$

$$m_n = 1.00867 u,$$

So, we would need $(m_d - m_p - m_n)c^2 = 0.0025u c^2$ of energy to split the deuteron into a proton and a neutron. That is equivalent to $\boxed{2.23 \text{ MeV}}$ of energy. This nuclear process is written as:



Problem 3: Photon Reabsorption

An excited particle emits a photon. Under which condition can this photon be reabsorbed?

- Suppose the excited particle is initially at rest with mass m_0 . After emitting a photon with energy Q , it has mass m'_0 and is moving at speed v .

By conservation of energy and momentum, we have:

$$\begin{aligned} m_0 c^2 &= Q + m'_0 \gamma(v) c^2 \\ 0 &= \frac{Q}{c} - m'_0 \gamma(v) v, \end{aligned} \tag{1}$$

which means the energy and momentum of the particle, after emitting the photon, are:

$$\begin{aligned} E' &= m_0 c^2 - Q \\ p' &= \frac{Q}{c}. \end{aligned}$$

Requiring the particle to be on-shell, it gives:

$$E'^2 - (p'c)^2 = (m_0 c^2 - Q)^2 - Q^2 = (m_0 c^2)^2 - 2m_0 c^2 Q = (m'_0 c^2)^2.$$

So,

$$\boxed{Q = Q_0 \left(1 - \frac{Q_0}{2m_0 c^2} \right) < Q_0,}$$

where

$$Q_0 \equiv (m_0 - m'_0) c^2.$$

The energy conservation in Equation 1 also requires:

$$\boxed{m'_0 = \frac{m_0 - \frac{Q}{c^2}}{\gamma(v)}}$$

Problem 4: Fixed-Target $\bar{p}$ Production

What is the minimal beam energy in a proton on proton fixed target experiment to produce anti-protons?

- The process can be written as: $p + p \longrightarrow p + p + p + \bar{p}$.
In the center of mass frame, $E = 2\gamma m_p c^2 = 4m_p c^2 \Rightarrow \gamma = 2; \beta = 0.866$.
Siwthing to the lab frame, $\beta = \frac{2*0.866}{1+0.866^2} = 0.990 \Rightarrow \gamma = 7$
So, we need $E = \gamma m_p c^2 \approx \boxed{6.57 \text{ GeV}}$ of beam energy in a proton on proton fixed target experiment to produce anti-protons.

Problem 5: Pion Decays

Assume the decay of a pion at rest into an electron and positron. How fast are the decau products?

- By conservation of momentum, the electron and positron must fly out in equal and opposite directions, which means $\gamma_{e^-} = \gamma_{e^+}$, and $E_{e^-} = E_{e^+} = \frac{1}{2}m_{\Pi}c^2 = 67.5 \text{ MeV}$.
 $E_{e^-} = \gamma_{e^-} m_e c^2 = \gamma_{e^-} (0.511 \text{ MeV}) \Rightarrow \gamma_{e^-} = 132 \Rightarrow \boxed{v = 0.99997 c}$.

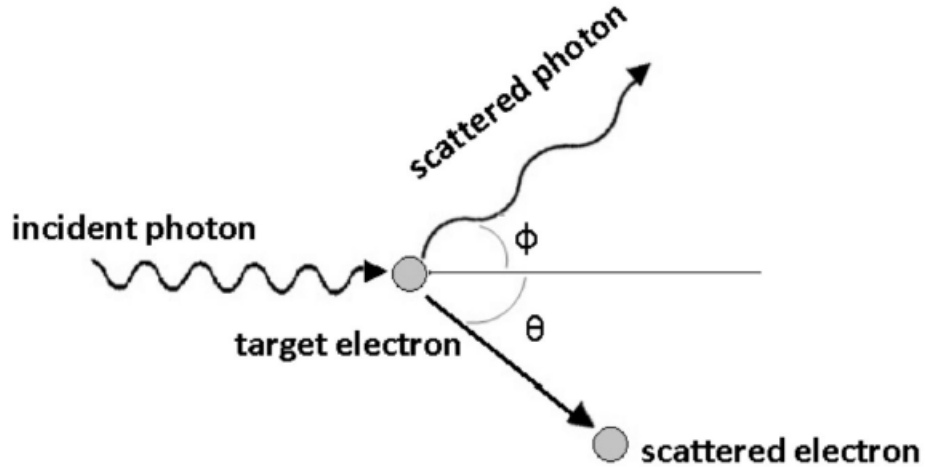
Problem 6: Fixed-Target Pion Production

What is the minimal beam energy of a proton colliding with a proton at rest to produce a $p + n + \Pi^+$?

- The process can be written as: $p + p \longrightarrow p + n + \Pi^+$.
In the center of mass frame, $E = 2\gamma m_p c^2 = (m_p + m_n + m_{\Pi^+})c^2 \Rightarrow$
 $\gamma = 938 + 940 = 1402 * 938 = 1.08; \beta = 0.37$.
Siwthing to the lab frame, $\beta = \frac{2*0.37}{1+0.37^2} = 0.65 \Rightarrow \gamma = 1.32$
So, we need $E = \gamma m_p c^2 \approx \boxed{1.24 \text{ GeV}}$ of beam energy in a proton on proton fixed target experiment to produce a $p+n+\Pi^+$.

Problem 7: Compton Effect

The energy of a photon is $E = h\nu = \frac{h}{\lambda}$. Calculate the change in the photon's wavelength.



- Conservation of 4-momenta:

$$\begin{aligned}
 p_\gamma + p_e &= p'_\gamma + p'_e \\
 (p_\gamma - p'_\gamma)^2 &= (p'_e - p_e)^2 \\
 p_\gamma^2 + p_\gamma'^2 - 2p_\gamma \cdot p'_\gamma &= p_e^2 + p_e'^2 - 2p_e \cdot p'_e
 \end{aligned}$$

We let $c = 1$ in the following derivation, and use the fact that $m_\gamma = p_\gamma^2 = 0$, $\vec{p}_e = 0$ and $p_\gamma = E_\gamma(1, \hat{r})$:

$$\begin{aligned}
 -2p_\gamma \cdot p'_\gamma &= 2m_e^2 - 2p_e \cdot p'_e \\
 -p_\gamma \cdot p'_\gamma &= m_e^2 - p_e \cdot p'_e \\
 -p_\gamma \cdot p'_\gamma &= m_e^2 - m_e E'_e \\
 -(E_\gamma E'_\gamma - \vec{p}_\gamma \cdot \vec{p}'_\gamma) &= m_e(m_e - E'_e) \\
 E_\gamma E'_\gamma(1 - \cos \phi) &= m_e(E'_e - m_e) \\
 E_\gamma E'_\gamma(1 - \cos \phi) &= m_e(E_\gamma - E'_\gamma)
 \end{aligned}$$

Since $E_\gamma = \frac{h}{\lambda}$,

$$\begin{aligned}
 \frac{h^2}{\lambda\lambda'}(1 - \cos \phi) &= m_e h \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right) \\
 \Delta\lambda = \lambda' - \lambda &= \boxed{\frac{h}{m_e}(1 - \cos \phi)}
 \end{aligned}$$

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